Basics of Digital Low-Dropout (LDO) Integrated Voltage Regulator

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Outline

Motivation

- Digital vs. Analog LDOs
- Key Specifications
- □ Interim Q/A
- □ State-of-the-Art Digital LDO Architectures
- □ Stability Analysis
- Concluding Remarks

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Concluding Remarks

Conventional Architecture



- □ Hard to integrate a buck on a chip
- Cores share a single voltage

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Integrated Low-Dropout (LDO) Voltage Regulators



Hard to integrate a buck on a chip
 LDO easy to integrate; no large passives
 Cores share a single voltage
 LDO can enable per-core voltage domain

Basics of Digital Low-Dropout (LDO) Integrated Voltage Regulator

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Analog LDO



□ Analog circuit-based feedback

V_g: gate voltage V_{in}: input voltage V_{out}: output voltage V_{ref}: reference voltage R_{load}: load resistance C_{out}: output capacitance

 I_{load} : load current I_{pwr} : power-FET current I_q : quiescent current I_{cap} : capacitor current

Digital LDO



□ Analog circuit-based feedback



Digital control, sandwiched by ADC and DAC

ADC: analog to digital converter, DAC: digital to analog converter

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Analog vs Digital LDOs

Analog LDO

- Pros
 - High bandwidth for fast transient response
 - High power supply rejection ratio (PSRR)
 - Small output ripple

Cons

- Complex/bulky analog design
- Limited scalability to low voltage
- Loop gain depends on operating voltage

Digital LDO

Pros

- No major analog components and synthesizable control
- Scales well for low-voltage operation
- Decouples loop gain from operating voltage
- Cons
 - Low bandwidth
 - Low PSRR
 - Large output ripple

Input Voltage

Analog LDO



- □ Limited V_{in} scalability
- \Box Typically, V_{in} > 0.6-0.7V
- A high-speed OP amplifier



- Better V_{in} scalability
- 0.5V or less

Power-FET's Source-Gate Voltage (V_{sg})

Analog LDO





□
$$V_{sg} = V_{in} - V_g$$
 between $V_{in} - V_{th}$ and $0V$
□ i.e., V_g is analog voltage

□ $V_{sg} = V_{in} - V_g$: *either* 0V or V_{in} □ i.e., V_g is digital voltage

Output Voltage

Analog LDO



- V_{out} = V_{in}-0.2V to ~0V
 Power FET is typically in saturation
- \Box If a load needs V_{in}-0.1V, we supply V_{in}



 $\Box V_{out} = V_{in} - 0.05V \text{ to } \sim 0V$

- Power FET is in linear to saturation
- Wider output voltage range

Dropout Voltage Requirement





- V_{in} set to the highest VDD requirement
 An LDO can be bypassed
- Less dropout voltage requirement
 Support wider output voltage

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- **Stability Analysis**

Concluding Remarks

Key Specifications

- Silicon area
- □ Input, reference, and output voltage $(V_{in}, V_{ref}, V_{out})$
- $\Box \quad Edge time (t_{edge})$
- □ Voltage droop and overshoot (V_{droop} , $V_{overshoot}$)
- $\square Response and settling time (t_{response,} t_{settle})$
- \Box Load, quiescent, power-FET, and capacitor current (I_{load} , I_q , I_{pwr} , I_{cap})
- **D** Peak current and power efficiency (CE_{peak} , PE_{peak})
- Dropout voltage (V_{dropout})
- Power supply rejection ratio (PSRR)
- □ Load regulation performance FoMs: ps FoM and pF FoM
- $\square Maximum and minimum load current (I_{load,max}, I_{load,min})$
- □ DAC and ADC number of bits (N_{DAC}, N_{ADC})
- **Dead-zone voltage** (V_{dz})
- $\Box \quad DAC \text{ step size } (V_{DAC,ss})$
- $\Box IR drop voltage (V_{IR})$

Silicon Area



65-nm example [Kim-VLSI19]

Active components scale better than passive with technology scaling

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Voltage Droop (V_{droop})



- V_{out}'s maximum *downward* deviation from V_{ref}
- V_{droop} is typically targeted to less than 10% of V_{out}
- $\begin{tabular}{ll} $ \Box$ & \Delta I_{load,max}$ can be different from $I_{load,max}$ end{tabular} \end{tabular}$

Edge Time (t_{edge})



- t_{edge} is a function of the clock period of a load
- □ 1 GHz core \rightarrow 1-ns t_{edge}
- □ 2 GHz core \rightarrow 0.5-ns t_{edge}

Response Time (t_{response})



- It is roughly proportional to feedback latency
- In synchronous control, it is proportional to t_{clk}
- In asynchronous control, it is proportional to circuit latency/delay

Settling Time (t_{settle})



- Time to take for V_{out} to settle within less than a small % of V_{ref} (e.g., +/-0.5%)
- The output can have a ripple

Voltage Overshoot (V_{overshoot})



- V_{out}'s maximum *upward* deviation from V_{ref}
- V_{overshoot} is typically targeted to less than 10% of V_{out}
- $\begin{tabular}{ll} $ $\Delta I_{load,max}$ is different from $I_{load,max}$ exactly $I_{load,$

Current and Power Efficiency (CE, PE)



Current Efficiency vs. *Peak* Current Efficiency



□ 99.9% CE_{peak} → 1% or less loss for I_{load} = **0.1**·**I**_{load,max} to I_{load,max}
 □ 99.99% CE_{peak} → 1% or less loss for I_{load} = **0.001**·**I_{load,max}** to I_{load,max}

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Dropout Voltage (V_{dropout})



Power Supply Rejection Ratio (PSRR)

$$PSRR = 20 \cdot \log \frac{\Delta V_{out}}{\Delta V_{in}} \quad [dB]$$



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ps Figure of Merit (ps-FoM)

$$Pseudo$$

$$response time$$

$$ps FoM = t_{response} \cdot \hat{I_q} = \left(\underbrace{C_{out} \cdot \Delta V_{out}}{\Delta I_{load, max}} \right) \cdot \left(\underbrace{I_q}{\Delta I_{load, max}} \right)$$

$$Pseudo current$$
efficiency

- Dynamic load regulation performance
- This FoM aims to capture the product of the LDO response time and current efficiency
- □ Smaller is better

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[Hazucha-JSSC05]

Caveats for the ps-FoM



- □ V_{in}, t_{edge}, and ΔI_{load,max} have a strong impact on FoM.
- □ ps FoM favors large $\Delta I_{load,max}$ (: $ps FoM \propto \Delta V_{out} / \Delta I_{load,max}^2$)
- Using a large output capacitor and proportionally increasing ΔI_{load,max} improves ps FoM in a same design
- $\square \quad Note \Delta I_{load,max}, not I_{load,max}$
- Ideal to compare designs having similar V_{in}, t_{edge}, and ΔI_{load,max}

pF Figure of Merit (pF-FoM)

$$pF \ FoM = I_q \left(\frac{\Delta V_{out}}{\Delta I_{load, max} \cdot V_{out}} \right) C_{out}$$
Normalized
voltage droop

- □ Alternative FoM for dynamic load regulation performance
- \Box How small the voltage droop is at the power (I_q) and area cost (C_{out})
- □ Smaller is better
- \square pF FoM works better to compare LDOs with different $\Delta I_{load,max}$
- Should use it for designs having similar V_{in} and t_{edge}

[Kim-JSSC17]

Maximum Load Current (I_{load,max})

Power FET unit current = I_u Maximum load current $(I_{load, max}) = \sum_{i=0}^{N_{DAC}-1} 2^i \cdot I_u$ Power FET unit current = I_u Maximum load current $(I_{load,max}) = \sum_{i=0}^{N_{DAC}-1} 2^i \cdot I_u$ $I_{load,max} = I_u \cdot (2^{N_{DAC}} - 1) \implies N_{DAC} = \log_2(\frac{I_{load,max}}{I_u} + 1)$

DAC Step Size (V_{DAC,SS})

Power FET unit current $= I_u$

If power FETs are sized in the power of 2

Maximum load current
$$(I_{load, max}) = \sum_{i=0}^{N_{DAC}-1} 2^{i} \cdot I_{u}$$

$$I_{load,max} = I_u \cdot (2^{N_{DAC}} - 1) \implies N_{DAC} = \log_2(\frac{I_{load,max}}{I_u} + 1)$$

Output voltage resolution (worst-case) $\dashrightarrow V_{DAC,SS} = I_u \cdot R_{load,max}$

Minimum Load Current (I_{load,min})

If power FETs are Power FET unit current $= I_{\mu}$ sized in the power of 2 Maximum load current $(I_{load, max}) = \sum_{i=1}^{N_{DAC}-1} 2^{i} \cdot I_{u}$ $I_{load, max} = I_u \cdot (2^{N_{DAC}} - 1) \implies N_{DAC} = \log_2(\frac{I_{load, max}}{I_u} + 1)$ r=0.01 if targeting Output voltage $resolution (worst-case) \dashrightarrow V_{DAC,SS} = I_u \cdot R_{load,max} \qquad V_{DAC,SS} = 1\% \text{ of } V_{out}$ $= I_u \cdot \frac{V_{out}}{I_{load,min}} = I_u \cdot \frac{V_{out}}{1/r} = r \cdot V_{out}$

Minimum Load Current (I_{load,min})

If power FETs are Power FET unit current $= I_{\mu}$ sized in the power of 2 Maximum load current $(I_{load,max}) = \sum_{i=1}^{N_{DAC}-1} 2^{i} \cdot I_{u}$ $I_{load,max} = I_u \cdot (2^{N_{DAC}} - 1) \implies N_{DAC} = \log_2(\frac{I_{load,max}}{I_u} + 1)$ r=0.01 if targeting Output voltage $resolution (worst-case) \dashrightarrow V_{DAC,SS} = I_u \cdot R_{load,max} \qquad V_{DAC,SS} = 1\% \text{ of } V_{out}$ $= I_u \cdot \frac{V_{out}}{I_{load,min}} = I_u \cdot \frac{V_{out}}{1/r} = r \cdot V_{out}$ $I_{load,min} = I_u \cdot \frac{V_{out}}{V_{DAC,SS}} = I_u \cdot \frac{V_{out}}{r \cdot V_{out}} = \frac{I_u}{r} \gg I_u$ for r=0.01

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Clock Frequency (f_{clk})



- Clock for a synchronous ADC and a controller
- □ High clock frequency (f_{clk}) improves $t_{response}$, t_{settle} , V_{droop} , and $V_{overshoot}$
- \Box High f_{clk} increases I_q, thus degrading CE

Output Ripple Size (V_{ripple})



- \Box High f_{clk} increases V_{ripple}
- Because a controller makes a correction before the previous correction is fully applied on a load [Nasir-TPE16]

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Output Ripple Size (V_{ripple})


IR Drop (V_{IR})



□ Parasitic resistance (R_{para}) can make $V_{out} \neq V_{out,far}$

- □ The ADC senses V_{out}, not V_{out,far}
- \Box This deviation, V_{IR}, grows with I_{load}

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Baseline Digital LDO Architecture



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Overview

Control Law	Triggering	Power FET	Digital/Analog
◀. Integral (I) feedback	1. Time-driven (synchronous)	1. Digital PFET	1. All-digital >> Baseline
 Multi-bit ADC Proportional and integral (PI) Feedforward Binary search 	 Adaptive sampling clock Event-driven (asynchronous) Self-triggered Domino triggering 	2. Digital NFET	2. Parallel PI3. Analog-assisted digital4. Hybrid digital and analog

Overview

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Integral Control



$$V_g[k] = i[k]$$
$$i[k] = i[k - 1] + err[k] \cdot K_I$$

 \Box i[k] = integration result □ err[k] = +1 or -1 \Box K₁ = integral gain \Box K_I = 1 for small V_{DAC.SS} Worst-case $t_{settle} = \frac{1}{f_{clk}} \cdot 2^{N_{DAC}}$

Multi-bit ADC

ADC number of bits: $N_{ADC} = \log_2(No.quantization \ level + 1)$



$$N_{ADC} = \log_2(1+1) = 2$$
 bit

Most common

[Kim-JSSC17]

- $N_{ADC} = \log_2(4+1) = 2.32 \ bit$
- For high-performance control
- \Box V_{ref} is between V_{ref1} and V_{ref2}
- Non-uniform quantization is common
- More silicon, power, and references

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Multi-bit ADC with Deadzone



Deadzone (V_{DZ}): a voltage range where a controller doesn't update its output No ripple if $0.5 \cdot V_{DAC,SS} < 0.5 \cdot V_{DZ}$ ($V_{DAC,SS}$: DAC step size)

[Kim-JSSC17]

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Multi-bit ADC-based Integral Control



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Integral Output,
$$i[k] = i[k - 1] + K_I \cdot err[k]$$

roportional Ouptut, $p[k] = K_P \cdot err[k]$
Controller Output, $V_g[k] = i[k] + p[k]$

- p[k] = instantaneous error-gain product (aka proportional control output)
- □ K_p set > 1 since it does not affect the DC error (i.e., err[k]=0 \rightarrow p[k]=0)
- \Box Increasing K_P:

P

- V_{droop} and t_{response} improvement
- Stability degradation

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[Kim-JSSC17]

Feedforward Control (aka Initialization)



- \Box Predict the needed amount of additional current (I_{cap}) by measuring V_{out} slope
- ff[k] = the output term of feedforward control
- LUT = look-up table, to avoid multiplication and division [Kim-VLSI18]

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48 of 79

Binary-Search Control



[Salem-JSSC18]

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4. Feedforward 5. Binary search	 Self-triggered Domino triggering 		and analog

Synchronous, aka Time-Driven (TD)



□ The worst-case response time $(t_{response})$ is ~2 clock cycle

- One cycle to wait for the next sampling edge
- One cycle to calculate and update V_g
- $\Box \quad F_s = 1 \text{ MHz} \rightarrow t_{\text{response}} = 2 \text{ } \mu \text{s}$

Adaptive LDO Sampling Clock

$$\begin{split} &V_{ripple} \cong \alpha \cdot I_{u} \cdot R_{load}, \text{, where } \alpha = 1 \text{ for } f_{clk} \ll \frac{I_{load,min}}{V_{out} \cdot C_{out}} \\ & \text{ if } I_{load} = I_{load,max}, \text{ then } f_{clk} \text{ can be large} \\ & \text{ if } I_{load} = I_{load,min}, \text{ then } f_{clk} \text{ should be small} \end{split}$$

\Box Adaptively change f_{clk} based on the present I_{load} level

[Nasir-TPE16]

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Asynchronous (a.k.a. Event-Driven)



Asynchronous Comparator



- Operating at low supply voltage
- □ Superior latency over synchronous (i.e., clocked) comparator
- □ Typically worse in the sampling rate and energy consumption per sampling

Integral Control in Event-Driven Triggering



Integration with non-uniform triggering requires time interval measurement

□ Also, it requires a hardware multiplier

[Kim-JSSC17]

Challenges in Event-Driven Trigger



- Need to measure the time interval between two events
- □ Need to have a multiplier
- \Box As V_{out} gets closer to V_{ref}, the event generation becomes slower
- □ Called a sticking problem. Bad for t_{settle}

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56 of 79

Self-Trigger



- Time measurement and multiplication are not needed
- □ No sticking problem \rightarrow Improving t_{settle}

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[Kim-VLSI18]

Domino Trigger





- A comparator output change triggers the next comparator
- □ Improve t_{response} and V_{droop}

[Kim-VLSI19]

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Digital PFET



Linear regionSaturation region $I_{load} \propto (V_{in} - V_g - V_{th}) \cdot (V_{in} - V_{out})$ $I_{load} \propto (V_{in} - V_g - V_{th})^{\alpha}$ Poor PSRR
Small V_dropoutPoor PSRR
Large V_dropout

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Digital NFET



Linear region

Saturation region

$$\begin{split} I_{load} \propto (V_{g,boosted} - V_{out} - V_{th}) \cdot (V_{in} - V_{out}) & I_{load} \propto (V_{g,boosted} - V_{out} - V_{out} - V_{out}) \\ & \text{Poor PSRR} & \text{No } V_{in} \text{: Good PSRR} \\ & \text{Small } V_{dropout} & \text{Large } V_{dropout} \end{split}$$

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 $(V_{th})^{\alpha}$

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Parallel Proportional and Integral



The output of P control is applied w/o waiting for I control
 Improve V_{droop} and t_{response}

[Kim-ISSCC17]

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Analog-Assisted Digital



- Digital and analog loop
- □ **Sharing** the same power FETs
- □ V_{out} droops $\rightarrow V_{SSB}$ droops $\rightarrow V_G$ droops \rightarrow power-FETs are more strongly turned on
- Performance depends on the g_m of power-FETs
- Weaker performance for low-tohigh I_{load} transition

[Huang-JSSC18]

Hybrid Digital and Analog



□ **Two sets** of power-FETs

- Coordination between two loops is needed
- Example: i) recovery from a large droop: use digital loop; ii) fine-grained output control & steady state: use analog loop
 [Liu-ISSCC19]

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State Space Representation: Error State





[Kim-JSSC17]

State Space Representation: Error State

Let's define it as A **Error state:** $err[k] = V_{ref} - V_{out}[k]$ V_{in} Output voltage: $V_{out}[k+1] = V_{out}[k] + I_{cap}[k] \cdot R_{load} \cdot (1 - e^{-T_s/(R_{load} \cdot C_{out})})$ $\therefore err[k+1] = V_{ref} - V_{out}[k+1]$ 1 DU Time $= V_{ref} - V_{out}[k] - I_{cap}[k] \cdot A$ interval $= err[k] - (I_{pwr}[k] - I_{load}[k]) \cdot A$ b/w two Vout samples, i.e., 1/f_{clk} Power FET current: $I_{pwr}[k] = i[k] \cdot I_u$ cap load Load current: $I_{load}[k] = \frac{V_{out}[k]}{R_{load}} = \frac{V_{ref} - err[k]}{R_{load}}$ R_{load} \$ Cout $\therefore err[k+1] = err[k] - \left(i[k] \cdot I_u - \frac{V_{ref} - err[k]}{R_{load}}\right) \cdot A$ $= \left(1 - \frac{1}{R_{load}} \cdot A\right) err[k] - (I_u \cdot A)i[k] + \frac{V_{ref}}{R_{load}} \cdot A$

[Kim-JSSC17]

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State Space Representation: Integral Control State

Vin LDO Ipwr=i[k]•lu Cout Cout Rload

Error state equation:
$$err[k+1] = \left(1 - \frac{1}{R_{load}} \cdot A\right) err[k] - (I_u \cdot A)i[k] + \frac{V_{ref}}{R_{load}} \cdot A$$

Integral control state $i[k+1] = i[k] + K_i \cdot err[k]$
equation:

$$\begin{bmatrix} err[k+1] \\ i[k+1] \end{bmatrix} = \begin{bmatrix} 1 - \frac{1}{R_{load}} \cdot A & -I_u \cdot A \\ K_i & 1 \end{bmatrix} \begin{bmatrix} err[k] \\ i[k] \end{bmatrix} + \begin{bmatrix} \frac{V_{ref}}{R_{load}} \cdot A \\ 0 \end{bmatrix}$$

[Kim-JSSC17]

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State Space Representation: Stability Condition

$$V_{in}$$

$$LDO$$

$$I_{pwr}=i[k] \bullet I_{u}$$

$$V_{out}$$

$$V_{out}$$

$$I_{cap}$$

$$V_{out}$$

$$I_{load}$$

$$K_{load}$$

$$V_{out}$$

$$I_{load}$$

[Kim-JSSC17]

State Space Representation: Results



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Concluding Remarks
Papers to See This Year

Session 25 Relevant Papers:

- 25.1: A Fully Synthesizable Distributed and Scalable All-Digital LDO in 10nm CMOS
- 25.2: A 480mA Output-Capacitor-Free Synthesizable Digital LDO Using CMP-Triggered Oscillator and Droop Detector with 99.99% Current Efficiency, 1.3ns Response Time and 9.8A/mm² Current Density
- 25.3: A 65nm Edge-Chasing Quantizer-Based Digital LDO Featuring 4.58ps-FoM and Side-Channel-Attack Resistance

Session 32 Relevant Papers:

- 32.4: A 0.4-to-1.2V 0.0057mm² 55fs-Transient-FoM Ring-Amplifier-Based Low-Dropout Regulator with Replica-Based PSR Enhancement
- 32.5: A Scalable and PCB-Friendly Daisy-Chain Approach to Parallelize LDO Regulators with 2.613% Current-Sharing Accuracy Using Dynamic Element Matching for Integrated Current Sensing

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