Fractional-*N***PLLs**

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Outline

- Summary of Integer-*N* PLL Limitations
- The Idea Behind Fractional-*N* PLLs
- Delta-Sigma Modulation Overview
- Fractional-*N* PLL Dynamics
- Spurious Tones in Fractional-*N* PLLs
- Fractional-*N* PLL Bandwidth Enhancement
- IC Implementation Examples (bonus slides: not presented)

A Typical Integer-NPLL





Have fundamental resolution/bandwidth/noise tradeoff

Lower f_{ref} (larger N) for a given f_{VCO}

- \Rightarrow finer tuning resolution
- \Rightarrow lower bandwidth (slower settling)
- \Rightarrow larger VCO noise contribution
- ⇒ larger in-band divider, reference, and charge pump noise contributions

Example: Bluetooth channel frequencies from a 19.68 MHz crystal



Problem: Must design loop bandwidth to be less than 4 kHz!

 \Rightarrow Can't meet the 200 µs settling requirement

 \Rightarrow Can't meet the Bluetooth phase noise requirements

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Idea: Periodically switch divider modulus between 122 and 123 so the *average modulus* is 122 + 51/492

- \Rightarrow Have desired VCO frequency (at least on average)
- \Rightarrow Loop bandwidth can be up to almost 2 MHz
- ⇒ Periodic switching causes large spurious tones at multiples of 40 kHz
- \Rightarrow At low frequencies the tones see little loop suppression

Observations From These Fractional-*N***PLLs**



- y[n] must be integer-valued with a sample-rate of f_{ref}
- The PLL lowpass filters y[n] measured at the VCO input, so $f_{VCO} = (N + \text{lowpass filtered version of } y[n]) \cdot f_{ref}$

Conclusion: Would like (y[n] - 51/492) to be zero-mean with most of its power outside the PLL's bandwidth



Idea: Use a *digital* $\Delta\Sigma$ *modulator* to quantize α such that the *quantization noise* is zero-mean and has most of its power outside the PLL bandwidth. Then

$$f_{VCO} = (122 + \alpha) \cdot f_{ref}$$

 \Rightarrow Have precise frequency control without excessive phase noise \Rightarrow Can also modulate VCO by changing α each reference period

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Quantizer Example

There are many types of $\Delta\Sigma$ modulators, but most involve feedback around uniform quantizers.



Quantizer Example

Such quantizers alone do not have well-behaved quantization noise:



In the 0-500 kHz band, the SNDR is only 14 dB

$\Delta \Sigma$ Modulator Example⁵

A *second-order* $\Delta\Sigma$ *modulator* using the same 9-level uniform quantizer:



Can show:

$$y[n] = x[n-2] + \underbrace{e_q[n] - 2e_q[n-1] + e_q[n-2]}_{\Delta\Sigma \text{ Modulator Quantization Noise (call it } e_{\Delta\Sigma}[n])}$$

Idea: The quantization noise, $e_q[n]$, is subjected to two zero-frequency zeros whereas the signal is just delayed

 \Rightarrow Quantization noise power is mostly at high frequencies

$\Delta \Sigma$ Modulator Example

Unlike the quantizer alone, the $\Delta\Sigma$ modulator has well-behaved quantization noise:



In the 0-500 kHz band, the SNDR is 84 dB with no spurious tones!

The $\Delta\Sigma$ Modulator Example in a Fractional-*N* PLL

Example 1 Revisited (k = 1 case):



PLL phase noise from the $\Delta\Sigma$ modulator meets the Bluetooth local oscillator requirements for the 50 (but not 500) kHz BW case

$\Delta \Sigma$ Modulator Limitations

• The $\Delta\Sigma$ quantization noise must go somewhere—ideally, above the PLL bandwidth (up to $f_{ref}/2$)

 \Rightarrow Phase noise spec places limit on PLL bandwidth

• Increasing the $\Delta\Sigma$ modulator order⁶ puts more of the quantization noise above the PLL bandwidth, but the PLL has limited filtering capabilities

 \Rightarrow Higher than 3rd-order $\Delta\Sigma$ modulation rarely used

- Must use dither, e.g., 1-bit pseudo-random LSB dither⁷⁻⁸, to eliminate spurious tones in *y*[*n*]
- $\Delta\Sigma$ modulators with 1-bit quantization are rarely used because they require $\max\{x[n]\} - \min\{x[n]\} < 1$ which limits the achievable output frequencies, and dithering does not eliminate spurious tones

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The PLL's phase noise, $\theta_{PLL}(t)$, depends on:

$$\theta_{ref}(t)$$
 = reference oscillator phase noise,

$$\theta_{VCO}(t) = \text{VCO phase noise,}$$

$$\theta_{div}(t)$$
 = divider phase noise,

 $\theta_{PFD}(t) = PFD$ phase noise,

- $i_n(t)$ = charge pump and loop filter noise (referred to the loop filter input), and
- $e_{\Delta\Sigma}[n] = \Delta\Sigma$ modulator quantization noise (Slide 12)

"Linearized" Fractional-N PLL Model⁹



Fractional-*N* and **Integer-***N* **PLL Model Differences**



Shaded region is the only difference between fractional-*N* and integer-*N* PLL linearized models

Usually, $N >> \alpha$, so fractional-*N* PLLs have the same loop equations and noise transfer functions (except for $\Delta\Sigma$ quantization noise) as integer-*N* PLLs

$\Delta\Sigma$ Modulator Quantization Noise Path



The PSD of $\varepsilon_{\Delta\Sigma}(t)$ for an L^{th} -order $\Delta\Sigma$ modulator:

$$S_{\varepsilon_{\Delta\Sigma}}(f) = \frac{\pi^2}{12f_{ref}} \left[2 \cdot \sin\left(\frac{\pi f}{f_{ref}}\right) \right]^{2(L-1)}$$



A Secondary Effect of the $\Delta\Sigma$ Modulator



Fractional-*N* PLL charge pump pulses are wider than integer-*N* charge pump pulses by charge pump pulses by $\left|\frac{1}{f_{VCO}}\sum_{k=0}^{n-1}e_{\Delta\Sigma}[k]\right|$ which has a variance of $1/(6f_{VCO}^2)$ or $1/(2f_{VCO}^2)$ for a $\Delta\Sigma$ modulator

order of 2 or 3, respectively

 \Rightarrow This increases the effect of charge pump noise on $\theta_{PLL}(t)$

Local oscillator PLL for a direct conversion Bluetooth receiver: <u>Requirements:</u>

- Crystal frequency: 19.68 MHz (other crystals are easier)
- PLL output frequencies: $f_{VCO} = 2.402 \text{ GHz} + k \text{ MHz}, k = 0, ..., 78$

 $f_{RW} > 40 \text{ kHz}$

- Loop bandwidth:
- Phase margin:
- Total PLL phase noise: $\leq -120 \text{ dBc/Hz}$ at $\geq 3 \text{ MHz}$ from f_{VCO}

 $PM > 60^{\circ}$

Assume:

Reference source, VCO, divider, PFD and charge pump have been designed and meet noise specifications provided the $\Delta\Sigma$ modulator and loop filter each contribute ≤ -130 dBc/Hz at 3 MHz from f_{VCO}

Design Tasks:

- Choose $\Delta\Sigma$ modulator and frequency plan
- Choose loop filter component values

Assume a 2nd-order $\Delta\Sigma$ modulator is sufficient (will verify later):



Frequency Plan:

- To get k = 0, 1, ..., or 18:
- To get k = 19, 21, ...,or 38:
- To get k = 39, 41, ...,or 57:
- To get k = 58, 60, ...,or 78:

set N = 122, $m = k \cdot 25 + 26$ set N = 123, $m = (k - 19) \cdot 25 + 9$ set N = 124, $m = (k - 39) \cdot 25 + 17$ set N = 125, $m = (k - 58) \cdot 25$ <u>Relevant loop equations:</u> (derived from linearized model¹⁶)

$$PM = \tan^{-1} \left(\frac{b-1}{2\sqrt{b}} \right), \quad \text{where } b = \frac{C_2}{C_1} + 1$$

$$f_{BW} = \frac{IK_{VCO}R}{2\pi N} \cdot \frac{b-1}{b}$$

$$RC_2 = \frac{\sqrt{b}}{2\pi f_{BW}}$$

$$S_{\theta_{PLL}}(f)\Big|_{\Delta\Sigma \text{ only}} \cong 10 \cdot \log \left\{ \frac{4\pi^2 b}{3f_{ref}} \left[\sin^2 \left(\frac{\pi f}{f_{ref}} \right) \right] \left(\frac{f_{BW}}{f} \right)^4 \right\} \quad \text{dBc/Hz},$$

$$\left(\text{valid for } f > \frac{C_1 + C_2}{2\pi RC_1 C_2} \right)$$

Calculations: (for $K_{VCO} = 200 \text{ MHz/V}, I = 200 \mu\text{A}$)

- 1) Choose b = 15 in *PM* equation to get $PM = 62^{\circ}$
- 2) Solve $S_{\theta_{PLL}} (3 \cdot 10^6 \text{ Hz}) \Big|_{\Delta \Sigma \text{ only}} = -130 \text{ dBc/Hz} \text{ to get } f_{BW} = 44 \text{ kHz}$
- 3) Use the f_{BW} equation to get $R = 915 \Omega$
- 4) Use the *RC*₂ and *b* equations to get $C_2 = 15.3$ nF and $C_1 = 1.1$ nF

Phase Noise:



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Reference Spurs and Fractional Spurs



PLL phase noise always contains spurious tones ("spurs"):

- > Have reference spurs at multiples of f_{ref} (like integer-*N* PLLs)
- > Have fractional spurs at multiples of αf_{ref} modulo f_{ref}

Fractional Spur Overview



 \Rightarrow fractional spurs at multiples of $\alpha f_{ref} = 1$ MHz

In general, a PLL lowpass filters most fractional spurs

- Effective only for spurs above PLL bandwidth
- > Spurs within PLL bandwidth are unfiltered (typ. > -60dBc)

 \Rightarrow design restrictions on α , f_{ref} , and the PLL bandwidth

Overview of Fractional Spur Mechanisms



Mechanism 1:

Nonlinear coupling of $v_{ref}(t)$ and $v_{vco}(t)$ (or $v_{div}(t)$) e.g., [Nth harmonic in $v_{ref}(t)$] $\times v_{vco}(t) \Rightarrow \alpha f_{ref}$ spur

Mechanism 2:

Nonlinear distortion of $\Delta\Sigma$ quantization noise by non-ideal analog circuits (little known because same spur frequencies as Mechanism 1)

Fractional Spur Mechanism 1

- Parasitic coupling of $v_{ref}(t)$ and $v_{vco}(t)$ (or $v_{div}(t)$) cause fractional spurs
- The greatest opportunities for such coupling occur in the PFD and CP because they process signals aligned to $v_{ref}(t)$ and $v_{div}(t)$:



Power supplies to the PFD and CP are the main coupling paths



- The $v_{ref}(t)$ edge causes ringing through the V_{DD} bond wire
- If ringing persists to the next $v_{div}(t)$ edge, the bottom flip-flop output is affected by $v_{ref}(t)$ as well as $v_{div}(t)$

 \Rightarrow nonlinear coupling of $v_{ref}(t)$ and $v_{div}(t)$

Solution: Offset Current to Separate PFD Edges¹³



- This separates edges of v_{ref} and v_{div} $\Rightarrow V_{DD}$ ringing has time to die out
- Similar to method presented in [10]
- But current source mismatches cause big reference spur!

A Simple Method to Reduce the Reference Spur

Can use a sampled loop filter to reduce the reference spur (and improve $\Delta\Sigma$ noise cancellation – details later)¹¹⁻¹⁴

Example:¹³



- Switch is open only when $i_{in} \neq 0$ (e.g., 25 ns per reference period) \Rightarrow no reference spur from current source mismatches
- Charge injection is impulsive so it sees $R \approx$ open circuit and is well-cancelled by the half-size dummy switches

Nonlinearities come from the analog circuits:

- Nonlinearities in the divider operate on $e_q[n]$
- Nonlinearities from other blocks operate on



If the nonlinearity applied to $e_q[n]$ or its running sum causes spurs, then the PLL's output signal contains spurs

Example: Effect of second-order distortion:



- Similar results occur with other types of non-linear distortion
- Similar results occur regardless of the type of $\Delta\Sigma$ modulator and dither used

 \Rightarrow The $\Delta\Sigma$ modulator is the root cause of the problem!
Plausibility Demonstration of Mechanism 2

- **Q:** How can nonlinear distortion create spurs from a "spur-free" sequence?
- A: The following example gives a simple plausibility demonstration

$$\underbrace{\dots,\pm 1,0,\pm 1,0,\pm 1,0,\pm 1,0,\dots}_{\text{white noise}} \longrightarrow ()^2 \longrightarrow \underbrace{\dots,1,0,1,0,1,0,1,0,\dots}_{\text{offset}+f_s/2 \text{ spur}}$$

The randomness is *sufficient* to ensure that the input is spur-free, but it is *insufficient* to ensure that its square is spur-free.

Details of Mechanism 2



where $\langle x \rangle$ = fractional part of *x*, and $\lfloor x \rfloor = x - \langle x \rangle$

Each periodic sequence has spurs. Randomness from the dither prevents spurs in y[n], but not in $y^k[n]$ for k > 1

 \Rightarrow must minimize analog circuit nonlinearity

Nonlinearity: Modulus-Dependent Divider Delays



- **Problem:** Modulus-dependent divider delays, i.e., error in $v_{div}(t)$ that depends upon y[n], can be a major source of nonlinearity
- **Solution:** Resynchronize divider output to nearest VCO edge or at least a higher-frequency edge within divider ¹⁵

Nonlinearity: Charge Pump Dead Zone

The "obvious" way to control the charge pump leads to a "dead zone" which causes nonlinearity:



Nonlinearity: Charge Pump Dead Zone

Observation: The charge in the $i_{CP}(t)$ pulse changes linearly with the phase difference between $v_{ref}(t)$ and $v_{div}(t)$ provided the current sources are on long enough to fully settle



Charge Pump Dead Zone Nonlinearity Solution¹⁶

Ideally, $i_{CP}(t) = 0$ when both CP current sources are on, so for each $i_{CP}(t)$ pulse, turn both on long enough to settle (at least T_D), with the $i_{CP}(t)$ pulse formed by their difference:



This eliminates the dead zone, but increases CP noise and mismatches cause a periodic ripple which adds to the reference spur

Charge Pump Dead Zone Nonlinearity Solution¹⁶



Nonlinearity arises from mismatched positive and negative currents:



The slope discontinuity implies nonlinear behavior

- **Q**: What's so hard about matching current sources?
- A: The voltages across the *p* and *n* current sources do not track and they span a wide range depending on the PLL frequency





(The T_D delay is to eliminate the dead zone as described previously)

A closer look at typical charge pump pulses:



Positive-Negative CP Mismatch Solutions

- ✓ Solution 1: Increase output impedance of charge pump current sources (but this requires headroom)
- ✓ Solution 2: Actively balance the charge pump current sources using replica bias and an op-amp feedback circuits¹⁷ (works well for narrow-band PLLs, but increases settling time)
- ✓ Solution 3: Modify PFD and charge pump so good matching is only required between like current sources¹⁸
- ✓ Solution 4: Use a charge pump offset (Slide 32) so only one charge pump current source carries phase information, and use a sampled loop filter (Slide 33) to avoid a large reference spur¹³

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<u>Fractional-NPLL Bandwidth Enhancement</u>

• IC Implementation Examples (bonus slides: not presented)



- ... but it also:
- reduces PLL settling time
- reduces sensitivity to VCO pulling
- enables an on-chip loop filter
- enables in-loop transmit modulation



Idea Behind $\Delta\Sigma$ Phase Noise Cancellation



A $\Delta\Sigma$ Phase Noise Canceling Fractional-*N* PLL

Use a wide PLL bandwidth but cancel the $\Delta\Sigma$ quantization noise prior to the loop filter ^{10, 12-14, 18-19}





The DAC cancels most of the $\Delta\Sigma$ quantization noise prior to loop filter so PLL bandwidth need not be small

Effect of DAC Gain Error



- DAC gain error degrades the phase noise cancellation
- Passive matching is sufficient in many cases when f_{ref} is large enough (e.g., \geq 35 MHz)
- Each halving of f_{ref} increases the phase noise from DAC gain error by $6(L-\frac{1}{2}) dB (L = \Delta \Sigma order)^{19}$
- Adaptive gain calibration can be used for $\log f_{ref}$ cases¹⁹

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A Direct Method of Adaptive Calibration²⁰



A Sign-LMS algorithm adjusts ΔI_{bias} until DAC gain is correct

- Term proportional to $e_{cp}[n]$ remains in v_{ctrl} if DAC cancellation is not perfect
- Since sgn{ $e_{cp}[n]$ }× $e_{cp}[n] = |e_{cp}[n]|$, integrator ramps up or down until ΔI_{bias} is adjusted properly

What's the Catch?



- v_{ctrl} can have a large DC component (it sets the VCO freq)
- Hence, the LMS loop contains a large $sgn\{e_{cp}[n]\}$ term
- But $sgn\{e_{cp}[n]\}$ contains large spurious tones
- To suppress the tones, the LMS loop BW must be <u>very</u> low
- \Rightarrow Very slow calibration settling, e.g., 1s in prior art

A Fast-Settling Adaptive Calibration Technique¹⁹

Idea: Split the VCO's varactor into 2 parallel halves; use the common-mode voltage to control the VCO and the differential-mode voltage to control the calibration loop



- VCO is controlled by its common-mode input voltage, but is insensitive to differential-mode voltage
- The differential-mode voltage is now available to independently control calibration loop

A Fast-Settling Adaptive Calibration Technique



- Two parallel half-sized loop filters and varactors create differential signal path for calibration loop
- Multiplication by ± 1 performed by current steering
- Calibration feedback loop is DC-free

The Calibration Loop Signal Path



The calibration loop is controlled by a differential-mode signal that has no DC component

- \Rightarrow Calibration signal does not have to be filtered out by the calibration loop
- \Rightarrow Can have a wide calibration loop BW!

The PLL Signal Path



The VCO output is insensitive to calibration signal \Rightarrow Calibration does not affect operation of PLL!

Example PLL IC 1: Block Diagram¹⁹



Output frequency: 2.4-2.5GHz; bandwidth: 730kHz Technology: 0.18µm CMOS; Supply Voltage: 1.8V

Example PLL IC 1: DAC Topology



Segmented dynamic element matching used to eliminate harmonic distortion from non-ideal DAC weights and pulse shapes ²¹

Example PLL IC 1: 1-bit DAC Circuit Details



 M_1 and M_2 used to minimize injection of channel charge into loop filter

Example PLL IC 1: Die Photograph



Example PLL IC 1: Measured Phase Noise

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Example PLL IC 1: Performance Details

Design Details			
Technology	TSMC 0.18 µm 1P6M CMOS		
Package and Die Area	$32 \text{ pin TQFN}, 2.2 \times 2.2 \text{ mm}^2$		
Reference Frequency	12 MHz		
Output Frequency	2.4 – 2.5 GHz		
Loop Bandwidth	> 730 kHz		
Measured Core Current Consum	nption (at 1.8V)		
VCO and Divider Buffer	6.9 mA	20.9 mA	
Divider	5.8 mA		
CP (dynamic biasing enabled)	2.7 mA		
Digital	0.5 mA		
DAC	3.6 mA		
Calibration	1.4 mA		
Measured Worst Case Integer-N Performance			
Phase Noise @ 100 kHz	-104 dBc/Hz		
Phase Noise @ 3 MHz	-126 dBc/Hz		
Reference Spur	-55 dBc		
Measured Worst Case Performance with DAC and Calibration Disabled			
Phase Noise @ 100 kHz	-88 dBc/Hz		
Phase Noise @ 3 MHz	-91 dBc/Hz		
Fractional Spur $(a) \ge 3$ MHz	-45 dBc		
Reference Spur	-52 dBc		
Measured Worst Case Performance with DAC and Calibration			
Enabled			
Phase Noise @ 100 kHz	-101 dBc/Hz		
Phase Noise @ 3 MHz	-124 dBc/Hz		
Fractional Spur (a) ≥ 3 MHz	-62 dBc		
Reference Spur	-53 dBc		

Example PLL IC 1: Measured Spur Performance

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A $\Delta\Sigma$ Modulator Replacement²² $\frac{18}{x_2[n]}$ $2^{16}\alpha \xrightarrow{19}$ 1st Quantization 2nd 16th Quantization Block Quantization $v[n] = \alpha + e_q[n]$ $x_{16}[n]$ $x_1 | n$ Block Block **Successive Requantizer** $x_d[n] \xrightarrow{20-d}$ 19-d $x_{d+1}[n]$ $\begin{cases} \text{even value, } & \text{if } x_d[n] = \text{even, } & (\text{discard LSB}) \\ \text{odd value, } & \text{if } x_d[n] = \text{odd.} \end{cases}$ $s_d^{\mathsf{T}}[n] =$ *d*th Quantization Block

- $2^{16}\alpha$ = integer (resolution of α is 2^{-16})
- Each quantization block divides by two and quantizes by one bit
- LSB of $s_d[n] + x_d[n]$ is zero so discarding it implements $\div 2$
- $e_q[n]$ is a linear combination of the $s_d[n]$ sequences
- $s_d[n]$ sequences must have properties desired of $e_q[n]$

Quantization block operation: $x_{d+1}[n] = \frac{1}{2} (x_d[n] + s_d[n])$

In this work, have designed $s_d[n]$ to:

- 1. Ensure that the bit-width of $x_{d+1}[n]$ is that of $x_d[n]$ minus one \Rightarrow parity of $s_d[n]$ must equal parity of $x_d[n]$ \Rightarrow magnitude of $s_d[n]$ must not be too large
- 2. Keep $t_d[n]$ bounded ($\Rightarrow 1^{st}$ -order shaped PSD)
- 3. Prevent spurs in $(s_d[n])^p$, p = 1, ..., 5, and $(t_d[n])^q$, q = 1, 2, 3(this requires $s_d[n] \in \{0, \pm 1, \pm 2, \pm 3\}$)

where $t_d[n]$ is the running sum of $s_d[n]$

Tradeoff: Achieving item 3 increases power of $s_d[n]$

Phase noise cancellation circumvents this problem

The Quantization Block Details



19-d Combinatorial Logic Truth Table:

$LSB \text{ of } x_d[n] = 0$		
<i>t</i> _d [<i>n</i> -1]	$r_d[n]$	$S_d[n]$
2	≥ 0 and ≤ 3	0
2	≤ -1 or ≥ 4	-2
1	$\leq -1 \text{ or } \geq 6$	0
1	≥ 0 and ≤ 5	-2
0	0 or 1	2
0	$\leq -1 \text{ or } \geq 4$	0
0	2 or 3	-2
-1	$\leq -1 \text{ or } \geq 6$	0
-1	≥ 0 and ≤ 5	2
-2	≥ 0 and ≤ 3	0
-2	≤ -1 or ≥ 4	2

LSB of $x_d[n] = 1$		
$t_d[n-1]$	$r_d[n]$	$S_d[n]$
2	$\leq -1 \text{ or } \geq 4$	-1
2	≥ 0 and ≤ 3	-3
1	≥ 1 and ≤ 3	1
1	$\leq -1 \text{ or } \geq 4$	-1
1	0	-3
0	≥ 0	1
0	≤ -1	-1
-1	≥ 1 and ≤ 3	-1
-1	≤ -1 or ≥ 4	1
-1	0	3
-2	$\leq -1 \text{ or } \geq 4$	1
-2	≥ 0 and ≤ 3	3

- $t_d[n]$ = running sum of $s_d[n]$
- $t_d[n]$ kept bounded $\Rightarrow 1^{st}$ -order PSD shape
- No spurs in $(s_d[n])^p$, p = 1, 2, ..., 5, and $(t_d[n])^q$, q = 1, 2, 3
- See [22] for the math

Example PLL IC 2: Block Diagram¹³



• $f_{VCO} \in 2.4 \text{ GHz ISM band}; f_{ref} = 12 \text{ MHz}; \text{PLL BW} = 975 \text{ kHz}$

- Phase noise cancellation with calibration (not shown) as in [19]
- Also contains a $\Delta\Sigma$ modulator for comparison
Example PLL IC 2: Meas. Close-In Fractional Spur



Example PLL IC 2: Measured Fractional Spur Levels

Comparison between $\Delta\Sigma$ Modulator and SR with and without offset current:



Example PLL IC 2: Measured Output Spectra



Example PLL IC 2: Performance Table

Design Details		
Technology	0.18 um 1P6M CMOS	
Package and die area	32 pin TQFN, $2.2 \times 2.2 \text{ mm}^2$	
Reference frequency, output frequency band	12 MHz, 2.4 – 2.5 GHz	
Measured loop bandwidth	975 kHz	
Measured Current Consumption ($V_{DD} = 1.8$ V)		
VCO and divider buffer	5.9 mA	
Divider	7.3 mA	
Charge pump, PFD, and buffers	8.6 mA	Core
Offset current pulse generator	0.6 mA	27.1 mA
Digital	1.9 mA	
DAC	2.8 mA	
Bandgap ref, crystal buffer, external buffer	9.8 mA	
Measured Fractional-N Performance		
Phase noise at 100 kHz	-98 dBc/Hz	
Phase noise at 3 MHz	-121 dBc/Hz	
Worst case inband fractional spur [†]	-64 dBc	
Worst case reference spur	-70 dBc	

[†]Over 4 IC copies each measured with 100 values of $0 < \alpha < 1$

Example PLL IC 2:Die Photograph



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